Conclusion

The initial adjoint-variable guess technique has been researched in this Note. All of the approximate initial adjoint variables can be obtained. The minimum-time orbital transfers for very low thrust are solved by using this technique, and the results are slightly better than those of the direct method. The numerical examples demonstrate that the technique is effective.

Acknowledgment

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Identifying Helicopter Control Gains Using Desired Output Histories

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Introduction

A NEW method is presented for synthesizing control feedback and feedforward gains using desired output histories for the given inputs. Conventional design techniques use parameterized design criteria, and the time response of the closed-loop system is evaluated after the control gains are computed. However, this technique directly uses pairs of input and output time histories to find control system parameters. The control gains are computed so that the closed-loop outputs are as close as possible to the user-specified ones for the given inputs and is viewed as an identification process. Among the various identification methods, this study uses a method based on the smoothing technique as proposed by Idan and Bryson.[1]

This method is computationally intensive, but it is possible to implement this algorithm with modern desktop computers. It can be applied to many cases: simple stability augmentation system design, command following logic design, gain scheduling, etc. As an example, the new method is applied to a helicopter control system design problem.

Design with an Identification Method

Identification problems are inherently nonlinear and must be solved iteratively. At each iteration step, the input and output data are smoothed using the current set of parameters. In the process of smoothing, adjoint variables, which are the sensitivities of the performance index to the state variables, are computed. The sensitivities of the performance index to the system parameters are then computed using the adjoint variables. The system parameters are updated using these sensitivities at the end of each iteration step. The process is repeated until the changes in the system parameters become negligible.

The smoothing procedure is described as follows. Given a discrete linear state-space model

$$\boldsymbol{x}(i+1) = \boldsymbol{\Phi}(\boldsymbol{\Theta})\boldsymbol{x}(i) + \boldsymbol{\Gamma}(\boldsymbol{\Theta})[\boldsymbol{u}_0(i) + \boldsymbol{w}(i)] \tag{1}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{x}(i) + \boldsymbol{D}[\boldsymbol{u}_0(i) + \boldsymbol{w}(i)] \tag{2}$$

and a pair of input and desired output histories

$$\boldsymbol{u}_0(i), \qquad i = 1, \ldots, N-1$$
$$\boldsymbol{y}_d(i), \qquad i = 1, \ldots, N$$

determine the control parameter set $\boldsymbol{\Theta}$ that minimizes the performance measure:

$$\begin{aligned} J = {} & \frac{1}{2}[\boldsymbol{x}(0) - \boldsymbol{x}_0]^T \boldsymbol{S}_0[\boldsymbol{x}(0) - \boldsymbol{x}_0] + \frac{1}{2}[\boldsymbol{x}(N) - \boldsymbol{x}_f]^T \\ & \times \boldsymbol{S}_f[\boldsymbol{x}(N) - \boldsymbol{x}_f] + \frac{1}{2}\sum_{i=0}^{N-1}\Big\{\boldsymbol{w}^T(i)\boldsymbol{R}\boldsymbol{w}(i) + [\boldsymbol{y}(i+1) \\ & - \boldsymbol{y}_d(i+1)]^T \boldsymbol{Q}[\boldsymbol{y}(i+1) - \boldsymbol{y}_d(i+1)]\Big\} \end{aligned} \tag{3}$$

where $\boldsymbol{Q}$, $\boldsymbol{R}$, $\boldsymbol{S}_0$, and $\boldsymbol{S}_f$ are weighting matrices; $\boldsymbol{w}$ is a perturbation input; and $\boldsymbol{x}_0$ and $\boldsymbol{x}_f$ are guesses for the initial and final conditions. This is the most general form of the performance index. If test data are used as the input and output pairs, the noise in the measurement can be handled by using this comprehensive form. For deterministic cases, setting the weightings on the unnecessary terms, such as $\boldsymbol{w}$, to large numbers makes the effects of those terms negligible in the final results. The system matrices ($\boldsymbol{\Phi}$ and $\boldsymbol{\Gamma}$) are functions of control parameters but are treated as constant matrices for each iteration step. The optimization problem is to minimize the following augmented performance index with respect to $\boldsymbol{x}$ and $\boldsymbol{w}$:

$$\bar{J} = J + \sum_{i=0}^{N-1} \boldsymbol{\lambda}^T(i+1)\{\boldsymbol{\Phi}(\boldsymbol{\Theta})\boldsymbol{x}(i) + \boldsymbol{\Gamma}(\boldsymbol{\Theta})[\boldsymbol{u}_0(i) + \boldsymbol{w}(i)] - \boldsymbol{x}(i+1)\} \tag{4}$$

where $\boldsymbol{\lambda}$ is the set of Lagrange multipliers for the state equations. For the solution to this minimization problem, see Idan and Bryson.[1]

The derivatives of the performance index with respect to the control parameters are computed as follows:

$$\frac{\mathrm{d}\bar{J}}{\mathrm{d}\boldsymbol{\Theta}} = \sum_{i=0}^{N} \boldsymbol{\lambda}^T(i+1)\left\{\frac{\partial \boldsymbol{\Phi}(\boldsymbol{K})}{\partial \boldsymbol{\Theta}}\boldsymbol{x}(i) + \frac{\partial \boldsymbol{\Gamma}(\boldsymbol{K})}{\partial \boldsymbol{\Theta}}[\boldsymbol{u}_0 + \boldsymbol{w}(i)]\right\} \tag{5}$$

Equation (5) is for only one pair of inputs and outputs. As the complexity of the system model increases, more than one pair of inputs and outputs may be needed to guarantee the uniqueness of the solution. For the case of multiple sets of inputs and outputs, the performance index is defined as

$$J = \sum_{s=1}^{n\,\mathrm{set}} w_s(s) J_s \tag{6}$$

where n set is the number of input/output pairs and $w_s(s)$ represents the weighting on the sth data set. For each iteration step, the program solves the optimization problem for each set independently. Once the derivatives are computed for all sets, the effective derivative is computed as

$$\frac{\mathrm{d}\bar{J}}{\mathrm{d}\boldsymbol{\Theta}} = \sum_{s=1}^{n\,\mathrm{set}} w_s(s)\frac{\mathrm{d}\bar{J}_s}{\mathrm{d}\boldsymbol{\Theta}} \tag{7}$$

At the end of each iteration step, the control parameters are updated using this derivative information. This study uses a conjugate gradient method to find the control parameters.

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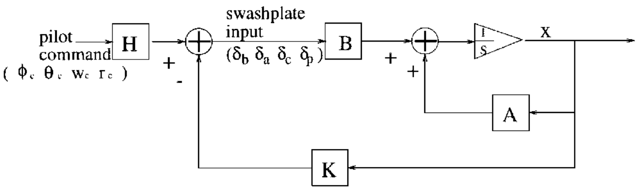


Fig. 1 Control system schematics.

The design process is summarized with the following steps:

1) Specify the desired input/output history pairs. Define the performance index (J) in quadratic form.

2) Guess the initial values for the parameters (control parameters $\boldsymbol{\Theta}$).

3) Find the state and output histories and the performance index for the given set of parameters. If the results are satisfactory, terminate the process. Otherwise, compute the adjoint variables and continue to the next step.

4) Evaluate $\mathrm{d}J/\mathrm{d}\boldsymbol{\Theta}$ using the adjoint variables. Update $\boldsymbol{\Theta}$ using this derivative information and go to step 3.

Design Example

The design method is applied to a rotorcraft control system design. The schematics of the control system are shown in Fig. 1. The system parameters are the feedback gain $\boldsymbol{K}$ and the feedforward gain $\boldsymbol{H}$, which is used to generate direct actuator inputs in response to pilot commands. The purpose of the design process is to find $\boldsymbol{K}$ and $\boldsymbol{H}$ so that the outputs are as close as possible to the user-specified outputs for the given pilot commands. This method finds $\boldsymbol{K}$ and $\boldsymbol{H}$ at the same time, which is different from many previous methods that compute them in two separate procedures.[2]

The rotorcraft model used here is an 8-degree-of-freedom (DOF) linear model,[†] derived from a nonlinear 27-DOF UH-60 simulation model that runs on FLIGHTLAB.[3] The open-loop model is unstable at hover and shows strong coupling between the longitudinal and lateral axes. The model is listed in Eq. (8), where the translational velocities are measured in units of 30 ft/s (9.14 m/s) and the angles and rates are measured in units of 20 deg and 20 deg/s, respectively. This scaling improved the convergence rate of the solution significantly. The controls are measured in degrees of swashplate movement and are ordered as longitudinal cyclic δ_b, lateral cyclic δ_a, collective angle δ_c, and tail rotor collective pitch δ_p.

One of the most important steps of the design procedure is to define the input/output pairs. For the case where the user designs a control system to make one vehicle behave like another vehicle, measurements of the inputs and outputs of the target vehicle during flight tests may be used. For the current example, the desired system response is generated based on an ideal model that meets the handling quality requirement. The base model shows the translational rate command[4] response type, which is the most stable response type. Thus, it is the preferred type in precision maneuvers under poor cues. The longitudinal, lateral, and collective stick controls are used to generate forward, lateral, and vertical translational velocities, respectively, whereas the pedal is used for yaw rate command. Four sets of time histories are generated with the base model. The inputs and initial conditions are as follows:

Set 1:

$$u(s) = \frac{0.8}{s^2 + 1.6s + 0.8}u_c(s), \qquad u_c(t) = 1$$

Set 2:

$$v(s) = \frac{0.8}{s^2 + 1.6s + 0.8}v_c(s), \qquad v_c(t) = 1$$

Set 3:

$$w(s) = (1/s + 1)w_c(s), \qquad w_c(t) = 1$$

Set 4:

$$r(s) = \frac{2.9}{s + 2.9}r_c(s), \qquad r_c(t) = 1$$

The default initial conditions are zero. The user needs to provide the initial guesses for the control gains $\boldsymbol{K}$ and $\boldsymbol{H}$. An linear quadratic regulator gain is used as an initial guess for $\boldsymbol{K}$. The initial guess for the feedforward matrix $\boldsymbol{H}$ is computed using the steady-state solution as[5]

$$\begin{bmatrix} \boldsymbol{X}_{\mathrm{ss}} \\ \boldsymbol{H}_{\mathrm{guess}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \mathbf{B} \\ \boldsymbol{C} & \mathbf{D} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \boldsymbol{I}_4 \end{bmatrix} \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{Bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \end{Bmatrix} = \begin{bmatrix} -0.0187 & 0.0014 & 0.0205 & -0.0173 & 0.0130 & -0.0039 & 0.0000 & -0.3735 \\ -0.0058 & -0.0631 & -0.0007 & -0.0142 & -0.0172 & 0.0067 & 0.3731 & 0.0014 \\ 0.0271 & -0.0051 & -0.3214 & -0.0029 & 0.0030 & 0.0281 & 0.0173 & -0.0293 \\ -0.1640 & -2.6689 & -0.0448 & -2.9213 & -1.4098 & 0.0168 & 0.0000 & 0.0000 \\ 0.0854 & 0.1414 & 0.3035 & 0.2320 & -0.5368 & -0.0660 & 0.0000 & -0.0001 \\ 0.1702 & 0.3348 & -0.0141 & -0.1286 & -0.1190 & -0.2846 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & -0.0036 & 0.0787 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9989 & 0.0463 & 0.0000 & 0.0000 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \end{Bmatrix}$$

$$+ \begin{bmatrix} 0.0198 & 0.0004 & 0.0146 & 0.0000 \\ -0.0012 & 0.0197 & -0.0021 & 0.0067 \\ 0.0005 & 0.0001 & -0.1757 & -0.0024 \\ -0.1249 & 2.1729 & -0.1169 & 0.2087 \\ -0.3589 & -0.0033 & 0.1802 & -0.0850 \\ 0.0049 & 0.1286 & 0.3731 & -0.1919 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \begin{Bmatrix} \delta_b \\ \delta_a \\ \delta_c \\ \delta_p \end{Bmatrix} \tag{8}$$

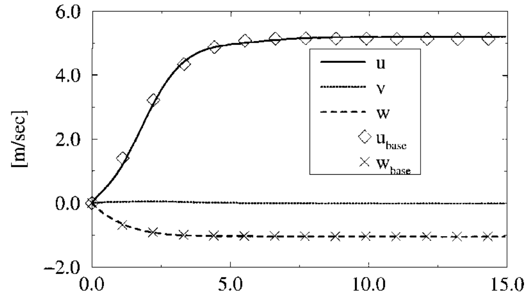
The $\boldsymbol{C}$ and $\boldsymbol{D}$ in the observation equation are set appropriately for the desired steady-state output vector of u, v, w, and r.

[†]The linear model is available from the author's Web site http://www.flightlab.com/~choi.

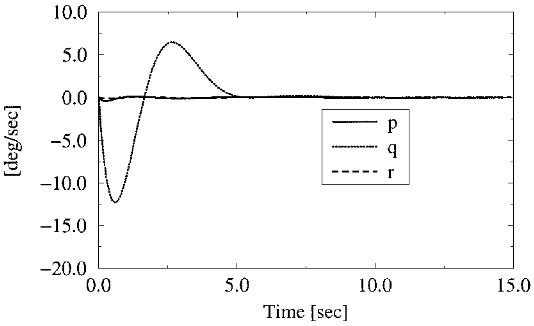
The gain matrices $\boldsymbol{H}$ and $\boldsymbol{K}$ are computed as

$$\boldsymbol{H} = \begin{bmatrix} 11.4274 & -0.9991 & -0.0076 & 2.9247 \\ 0.4293 & 5.0924 & 0.8072 & 1.0563 \\ -0.0593 & -0.2245 & -5.3610 & 0.1033 \\ 0.7600 & 2.5943 & -10.1460 & -11.4807 \end{bmatrix} \tag{10}$$

$$\boldsymbol{K} = \begin{bmatrix} 10.8358 & -0.8842 & -0.5380 & -1.1193 & -4.0450 & 2.5219 & -0.5852 & -9.1572 \\ 0.4481 & 3.7913 & 0.3816 & 1.3325 & -0.7996 & 1.1609 & 3.8849 & -0.3396 \\ -0.2041 & -0.1537 & -3.5639 & -0.0987 & -0.0227 & -0.0796 & -0.2615 & 0.2123 \\ -0.4035 & 0.1604 & -6.8850 & 1.1642 & -0.2847 & -10.4001 & 1.7214 & -0.2575 \end{bmatrix} \tag{11}$$



a) Translational rates



b) Angular rates

Fig. 2 Response of the closed-loop system to a combinational speed command.

The resultant closed-loop system is tested for a slow takeoff procedure. The forward and vertical velocities are commanded to 10 kn (5.14 m/s) and 2 kn (1.03 m/s), respectively. It should be noted that this test case is rather generic and not directly related to the base responses that were used in the design process. Figure 2a shows the resultant translational velocities. The primary axis values matched well with the base model response (u_{base}, w_{base}) while the off-primary axis response is decoupled. Figure 2b shows the angular rate responses. Deviation in the pitch rate is significantly large compared to the roll and yaw responses, which is natural for this type of maneuvering. Figure 2b also shows that the system is well decoupled, which results in the negligible roll and yaw rates.

Conclusion

A new design method that utilizes the desired time history and an identification algorithm is proposed. The technique was applied to designing a helicopter control system for translational velocity command with zero yaw rate. The feedback and feedforward gains were found to make the system behave like a reference model. The resultant closed-loop system was tested for a slow takeoff case. The system response of the primary channel was very close to that of the reference model, and the off-axis response was negligible.

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Solving Control Allocation Problems Using Semidefinite Programming

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I. Introduction

THE capabilities of modern combat and civilian aircraft keep increasing. In particular, modern-day aircraft have many available control surfaces and thrust vectoring capabilities that offer significant advantages over conventional architectures based on three control surfaces only, including reduced electromagnetic signature, tailless designs, energy-efficient maneuvering, and most importantly, much needed redundancy in case of battle damage. The trend toward the presence of multiple actuators in modern aircraft

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